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δ-CONTINUOUS AND IRRESOLUTE MAPS IN PYTHAGOREAN FUZZY NANO TOPOLOGICAL SPACES AND ITS APPLICATION

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Abstract: In this paper, we develop the concept of Pythagorean fuzzy nano (resp. δ, $\delta\mathcal{P}$, $\delta\mathcal{S}$, $\delta\alpha$ & $\delta\beta$ or e^*)-continuity in Pythagorean fuzzy nano topological spaces and specialize some of their basic properties with examples. Also, we discuss about properties and characterization of Pythagorean fuzzy irresolute maps and application of Multiple Criteria Decision Making (MCDM) techniques to the real-world problem using a proposed similarity measure in Pythagorean fuzzy nano topological spaces.

Keywords and Phrases: $\mathcal{PF}\mathfrak{N}\delta Cts$, $\mathcal{PF}\mathfrak{N}\delta\mathcal{S}Cts$, $\mathcal{PF}\mathfrak{N}\delta Irr$, $\mathcal{PF}\mathfrak{N}\delta\mathcal{S}Irr$ and Zhang similarity measure.

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1. Introduction

In 1965, Zadeh [39] familiarized the concept of fuzzy set which has several applications in decision theory, artificial intelligence, operations research, expert systems, computer science, data analytics, pattern recognition, management science and robotics. In 1968, Chang and Warren [14, 33] defined fuzzy topological spaces, the basic philosophies of topology such as open set, closed set, neighbourhood, interior set, closure, continuity, compactness to fuzzy topological spaces (FTS). Applications of fuzzy sets were studied [1, 13, 25, 30]. Later numerous fuzzy topological spaces raised which have unique properties. In 1997, Dogan Coker [9, 15, 19] introduced Intuitionistic fuzzy topological spaces and studied its continuity and compactness. Intuitionistic fuzzy sets have many applications [27, 30] and also flagged approach to study Pythagorean fuzzy sets. In both the sets membership and non-membership are incorporated in a different way. In Intuitionistic fuzzy set the membership μ and non-membership λ are incorporated in such a way that $\mu + \lambda \leq 1$ where as in Pythagorean fuzzy set it is $\mu^2 + \lambda^2 \leq 1$. In 2013, Yager [36] introduced the non-standard fuzzy sets called Pythagorean fuzzy sets in comparison with Intuitionistic fuzzy sets. He gave the basic definition of Pythagorean fuzzy set (PFS) and its application in decision making [3, 37, 38]. PFS has its applications in career placements based on academic performance [20], selection of mask during COVID-19 pandemic using Pythagorean TOPSIS technique [24], etc. Later Murat et.al [18] introduced the conception of Pythagorean fuzzy topological space ($PFTS$) by provoking from the conviction of FTS [16, 17, 23]. He defined Pythagorean fuzzy continuous function between $PFTS$.

Saha [26] defined δ-open sets in fuzzy topological spaces. In 2019, Acikgoz and Esenbel [2] defined neutrosophic soft δ-topology. Aranganayagi et al., Surendra et al. and Vadivel et al. [7, 8, 28, 29, 31, 32] introduced δ-open sets in neutrosophic, neutrosophic soft, neutrosophic hypersoft and neutrosophic nano topological spaces and studied its maps and separation axioms.

Similarity measure is a significant means for measuring the uncertain information. The fuzzy similarity measure is a measure that depicts the closeness (difference) among fuzzy sets. Zhang [40] proposed the Pythagorean fuzzy similarity measures for dealing the multi-attribute decision-making problems. Peng et al. [21] proposed the many new distance measures and similarity measures for dealing the issues of pattern recognition, medical diagnosis and clustering analysis, and discussed their transformation relations. Wei and Wei [34] presented some Pythagorean fuzzy cosine function for dealing with the decision-making problems. However, some existing similarity measures/distance measures cannot obey the third or fourth axiom, and also have no power to differentiate positive difference and negative difference or deal with the division by the zero problem. Due to the above counter-intuitive phenomena [21, 34, 40] of the existing similarity measures of PFS's, they may be hard for DM's to choose convincible or optimal alternatives. As a consequence, the goal of this paper is to deal with the above issue by proposing a novel similarity measure for Pythagorean fuzzy set, which can be without counter intuitive phenomena.

1.1 Research Gap

No investigation on some stronger and weaker forms of Pythagorean fuzzy continuous and irresolute maps such as Pythagorean fuzzy nano δ open map, Pythagorean fuzzy nano δ-semi open map, Pythagorean fuzzy nano δ-pre open map, Pythagorean fuzzy nano $\delta\alpha$ open map and Pythagorean fuzzy nano $\delta\beta$ open maps on Pythagorean fuzzy nano topological space has been reported in the Pythagorean fuzzy nano literature.

This leads to encompass the notion of $\mathcal{PFN}ts$ by introducing Pythagorean fuzzy nano δ (resp. $\delta\alpha$, $\delta\mathcal{S}$, $\delta\mathcal{P}$ & $\delta\beta$ or e^*)-continuous and discuss its properties. Also, we introduce the concept of Pythagorean fuzzy nano irresoluteness called Pythagorean fuzzy nano (resp. δ, $\delta\mathcal{P}$, $\delta\mathcal{S}$, $\delta\alpha$ and $\delta\beta$)-irresolute maps by using $\mathcal{PFNS}os$ (resp. $\mathcal{PFN}\delta os$, $\mathcal{PFN}\delta\mathcal{P}os$, $\mathcal{PFN}\delta\mathcal{S}os$, $\mathcal{PFN}\delta\alpha os$ and $\mathcal{PFN}\delta\beta os$)'s and study some of their basic properties. This definition enables us to obtain conditions under which maps and inverse maps preserve respective open sets.

2. Preliminaries

We recall some basic notions of fuzzy sets, IFS's and pfs's.

Definition 2.1. [39] *Let X be a nonempty set. A fuzzy set A in X is characterized by a membership function $\mu_A : X \to [0, 1]$. That is:*

$$\mu_A(x) = \begin{cases} 1, & if \quad x \in X \\ 0, & if \quad x \notin X \\ (0,1) & if\ x\ is\ partly\ in\ X. \end{cases}$$

Alternatively, a fuzzy set A in X is an object having the form $A = \{< x, \mu_A(x) > |x \in X\}$ or $A = \left\{ \left\langle \frac{\mu_A(x)}{x} \right\rangle |x \in X \right\}$, where the function $\mu_A(x) : X \to [0, 1]$ defines the degree of membership of the element, $x \in X$.

The closer the membership value $\mu_A(x)$ to 1, the more x belongs to A, where the grades 1 and 0 represent full membership and full nonmembership. Fuzzy set is a collection of objects with graded membership, that is, having degree of membership. Fuzzy set is an extension of the classical notion of set. In classical set theory, the membership of elements in a set is assessed in a binary terms according to a bivalent condition; an element either belongs or does not belong to the set. Classical bivalent sets are in fuzzy set theory called crisp sets. Fuzzy sets are generalized classical sets, since the indicator function of classical sets is special cases of the membership functions of fuzzy sets, if the latter only take values 0 or 1. Fuzzy sets theory permits the gradual assessment of the membership of element in a set; this is described with the aid of a membership function valued in the real unit interval $[0, 1]$.

Let us consider two examples:

(i) all employees of XYZ who are over $1.8m$ in height; (ii) all employees of XYZ who are tall. The first example is a classical set with a universe (all XYZ employees) and a membership rule that divides the universe into members (those over $1.8m$) and nonmembers. The second example is a fuzzy set, because some employees are definitely in the set and some are definitely not in the set, but some are borderline.

This distinction between the ins, the outs, and the borderline is made more exact by the membership function, μ. If we return to our second example and let A represent the fuzzy set of all tall employees and x represent a member of the universe X (i.e. all employees), then $\mu_A(x)$ would be $\mu_A(x) = 1$ if x is definitely tall or $\mu_A(x) = 0$ if x is definitely not tall or $0 < \mu_A(x) < 1$ for borderline cases.

Definition 2.2. [9, 10, 11, 12] *Let a nonempty set X be fixed. An IFS A in X is an object having the form: $A = \{< x, \mu_A(x), \lambda_A(x) > |x \in X\}$ or $A = \left\{\left\langle\frac{\mu_A(x),\lambda_A(x)}{x}\right\rangle |x \in X\right\}$, where the functions $\mu_A(x) : X \to [0, 1]$ and $\lambda_A(x) : X \to [0, 1]$ define the degree of membership and the degree of nonmembership, respectively, of the element $x \in X$ to A, which is a subset of X, and for every $x \in X$: $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$. For each A in X: $\pi_A(x) = 1 - \mu_A(x) - \lambda_A(x)$ is the intuitionistic fuzzy set index or hesitation margin of x in X. The hesitation margin $\pi_A(x)$ is the degree of nondeterminacy of $x \in X$ to the set A and $\pi_A(x) \in [0, 1]$. The hesitation margin is the function that expresses lack of knowledge of whether $x \in X$ or $x \notin X$. Thus: $\mu_A(x) + \lambda_A(x) + \pi_A(x) = 1$.*

Definition 2.3. [35, 36, 38] *Let a non empty set X be a universal set. Then, a Pythagorean fuzzy set A, which is a set of ordered pairs over X, is defined by the following: $A = \{< x, \mu_A(x), \lambda_A(x) > |x \in X\}$ or $A = \left\{\left\langle\frac{\mu_A(x),\lambda_A(x)}{x}\right\rangle |x \in X\right\}$, where the functions $\mu_A(x) : X \to [0, 1]$ and $\lambda_A(x) : X \to [0, 1]$ define the degree of membership and the degree of nonmembership, respectively, of the element $x \in X$ to A, which is a subset of X, and for every $x \in X$, $0 \leq (\mu_A(x))^2 + (\lambda_A(x))^2 \leq 1$. Supposing $(\mu_A(x))^2 + (\lambda_A(x))^2 \leq 1$, then there is a degree of indeterminacy of*

$x \in X$ to A defined by $\pi_A(x) = \sqrt{1 - [(\mu_A(x))^2 + (\lambda_A(x))^2]}$ and $\pi_A(x) \in [0, 1]$. In what follows, $(\mu_A(x))^2 + (\lambda_A(x))^2 + (\pi_A(x))^2 = 1$. Otherwise, $\pi_A(x) = 0$ whenever $(\mu_A(x))^2 + (\lambda_A(x))^2 = 1$. We denote the set of all PFS's over X by $pfs(X)$.

Definition 2.4. [38] *Let A and B be pfs's of the forms $A = \{< a, \mu_A(a), \lambda_A(a) > |a \in X\}$ and $B = \{< a, \mu_B(a), \lambda_B(a) > |a \in X\}$. Then*

(i) $A \subseteq B$ *if and only if* $\mu_A(a) \leq \mu_B(a)$ *and* $\lambda_A(a) \geq \lambda_B(a)$ *for all* $a \in X$.

(ii) $A = B$ *if and only if* $A \subseteq B$ *and* $B \subseteq A$.

(iii) $\bar{A} = \{< a, \lambda_A(a), \mu_A(a) > |a \in X\}$, *where* $\bar{A}$ *is the complement of* A.

(iv) $A \cap B = \{< a, \mu_A(a) \wedge \mu_B(a), \lambda_A(a) \vee \lambda_B(a) > |a \in X\}$.

(v) $A \cup B = \{< a, \mu_A(a) \vee \mu_B(a), \lambda_A(a) \wedge \lambda_B(a) > |a \in X\}$.

(vi) $0_P = \{< a, 0, 1 > |a \in X\}$ *and* $1_P = \{< a, 1, 0 > |a \in X\}$.

(vii) $\bar{1_P} = 0_P$ *and* $\bar{0_P} = 1_P$.

Definition 2.5. [4] *Let U be a non-empty set and R be an equivalence relation on U. Let A be a Pythagorean fuzzy set in U with the membership function $\mu_A(x)$ and non membership function $\lambda_A(x)$, $\forall\ x \in U$. The Pythagorean fuzzy nano lower, Pythagorean fuzzy nano upper approximation and Pythagorean fuzzy nano boundary of A in the approximation (U, R) denoted by $\underline{\mathcal{PF}\mathfrak{N}}(A), \overline{\mathcal{PF}\mathfrak{N}}(A)$ and $B_{\mathcal{PF}\mathfrak{N}}(A)$ are respectively defined as follows:*

(i) $\underline{\mathcal{PF}\mathfrak{N}}(A) = \left\{ \langle x, \mu_{\underline{R}(A)}(x), \lambda_{\overline{R}(A)}(x) \rangle / y \in [x]_R, x \in U \right\}$

(ii) $\overline{\mathcal{PF}\mathfrak{N}}(A) = \left\{ \langle x, \mu_{\overline{R}(A)}(x), \lambda_{\underline{R}(A)}(x) \rangle / y \in [x]_R, x \in U \right\}$

(iii) $B_{\mathcal{PF}\mathfrak{N}}(A) = \overline{\mathcal{PF}\mathfrak{N}}(A) - \underline{\mathcal{PF}\mathfrak{N}}(A)$

where $\mu_{\underline{R}(A)}(x) = \bigwedge_{y \in [x]_R} \mu_A(y)$
$\lambda_{\underline{R}(A)}(x) = \bigwedge_{y \in [x]_R} \lambda_A(y)$,
$\mu_{\overline{R}(A)}(x) = \bigvee_{y \in [x]_R} \mu_A(y)$,
$\lambda_{\overline{R}(A)}(x) = \bigvee_{y \in [x]_R} \lambda_A(y)$.

Definition 2.6. [4] *Let U be an universe of discourse, R be an equivalence relation on U and A be a Pythagorean fuzzy set in U and if the collection $\tau_{\mathcal{R}}(A) = \left\{0_{\mathcal{P}}, 1_{\mathcal{P}}, \underline{\mathcal{PF}\mathfrak{N}}(A), \overline{\mathcal{PF}\mathfrak{N}}(A), B_{\mathcal{PF}\mathfrak{N}}(A)\right\}$ forms a topology then it is said to be a*

Pythagorean fuzzy nano topology. We call $(U, \tau_{\mathcal{R}}(A))$ (or simply U) as the Pythagorean fuzzy nano topological space. The elements of $\tau_{\mathcal{R}}(A)$ are called Pythagorean fuzzy nano open (briefly, $\mathcal{PF}\mathfrak{N}o$) sets.

Remark 2.1. [4] *$[\tau_{\mathcal{R}}(A)]^c$ is called the dual fuzzy nano topology of $\tau_{\mathcal{R}}(A)$. Elements of $[\tau_{\mathcal{R}}(A)]^c$ are called Pythagorean fuzzy nano closed (briefly, $\mathcal{PF}\mathfrak{N}c$) sets. Thus, we note that a Pythagorean fuzzy set G of U is Pythagorean fuzzy nano closed in $\tau_{\mathcal{R}}(A)$ if and only if $1_P - G$ is Pythagorean fuzzy nano open in $\tau_{\mathcal{R}}(A)$.*

Definition 2.7. [4, 5] *Let $(U, \tau_{\mathcal{P}}(A))$ be a $\mathcal{PF}\mathfrak{N}ts$ with respect to A where A is a Pythagorean fuzzy subset of U. Let S be a Pythagorean fuzzy subset of U. Then Pythagorean fuzzy nano*

(i) interior of S (briefly, $\mathcal{PF}\mathfrak{N}int(S)$) is defined by $\mathcal{PF}\mathfrak{N}int(S) = \cup\{I : I \leq S \ \& \ I$ is a $\mathcal{PF}\mathfrak{N}o$ set in $U\}$.

(ii) closure of S (briefly, $\mathcal{PF}\mathfrak{N}cl(S)$) is defined by $\mathcal{PF}\mathfrak{N}cl(S) = \cap\{A : S \leq A \ \& \ A$ is a $\mathcal{PF}\mathfrak{N}c$ set in $U\}$.

(iii) regular open (briefly, $\mathcal{PF}\mathfrak{N}ro$) set if $S = \mathcal{PF}\mathfrak{N}int(\mathcal{PF}\mathfrak{N}cl(S))$.

(iv) regular closed (briefly, $\mathcal{PF}\mathfrak{N}rc$) set if $S = \mathcal{PF}\mathfrak{N}cl(\mathcal{PF}\mathfrak{N}int(S))$.

Definition 2.8. [6] *Let $(U_1, \tau_P(A_1))$ and $(U_2, \tau_P(A_2))$ be two $\mathcal{PF}\mathfrak{N}t$. Then a function $h_P : U_1 \to U_2$ is said to be a Pythagorean fuzzy nano continuous (briefly, $\mathcal{PF}\mathfrak{N}Cts$) function if $h_P^{-1}(G)$ is $\mathcal{PF}\mathfrak{N}o$ set in U_1 for all $\mathcal{PF}\mathfrak{N}o$ set G in U_2.*

Definition 2.9. [22] *Let M, N and O be three pfs's on X. A similarity measure $S(M, N)$ is mapping $S : pfs(X) \times pfs(X) \to [0, 1]$, possessing the following properties:*

(S1) $0 \leq S(M, N) \leq 1$;

(S2) $S(M, N) = S(N, M)$;

(S3) $S(M, N) = 1$ *iff* $M = N$;

(S4) $S(M, M^c) = 0$ *iff M is a crisp set;*

(S5) If $M \subseteq N \subseteq O$, then $S(M, O) \leq S(M, N)$ and $S(M, O) \leq S(N, O)$.

Let $X = x_1, x_2, \ldots, x_n$ be a finite universe of discourse, and A and B be two PFS's in X, in which $A = \{< x_i, \mu_A(x_i), \lambda_A(x_i) > | x_i \in X\}$ and $B = \{< x_i, \mu_B(x_i), \lambda_B(x_i) > | x_i \in X\}$.

Using the similarity measure in section 4, we have the Zhang [40] similarity measure are defined by

$$S_Z(A,B) = \frac{1}{2}\sum_{i=1}^{n} \frac{(|\mu_A^2(x_i)-\nu_B^2(x_i)|+|\nu_A^2(x_i)-\mu_B^2(x_i)|+|\pi_A^2(x_i)-\pi_B^2(x_i)|)}{|\mu_A^2(x_i)-\mu_B^2(x_i)|+|\nu_A^2(x_i)-\nu_B^2(x_i)|+|\pi_A^2(x_i)-\pi_B^2(x_i)|+|\mu_A^2(x_i)-\nu_B^2(x_i)|+|\nu_A^2(x_i)-\mu_B^2(x_i)|+|\pi_A^2(x_i)-\pi_B^2(x_i)|}.$$

3. Pythagorean fuzzy nano δ (resp. δ pre, δ semi, $\delta\alpha$ and $\delta\beta$)-continuous mappings

In this section, we introduce Pythagorean fuzzy nano δ (resp. δ pre, δ semi, $\delta\alpha$ and $\delta\beta$)-continuous mappings and discuss some of their properties.

Definition 3.1. *Let $(U, \tau_{\mathcal{P}}(A))$ be a $\mathcal{PFN}ts$ with respect to A where A is a pfs of U. Let S be a pfs of U. Then*

(i) Pythagorean fuzzy nano δ interior of S (briefly, $\mathcal{PFN}\delta int(S)$) is defined by $\mathcal{PFN}\delta int(S) = \cup\{I : I \subseteq S \ \& \ I$ is a $\mathcal{PFN}ro$ set in $U\}$.

(ii) Pythagorean fuzzy nano δ closure of S (briefly, $\mathcal{PFN}\delta cl(S)$) is defined by $\mathcal{PFN}\delta cl(S) = \cap\{A : S \subseteq A \ \& \ A$ is a $\mathcal{PFN}rc$ set in $U\}$.

Definition 3.2. *Let $(U, \tau_{\mathcal{P}}(A))$ be a $\mathcal{PFN}ts$ with respect to A where A is a pfs of U. Then a $\mathcal{PF}s$ S in U is said to be Pythagorean:*

(i) fuzzy nano δ-open set (briefly, $\mathcal{PFN}\delta os$) if $S = \mathcal{PFN}\delta int(S)$.

(ii) fuzzy nano $\delta\mathcal{P}$-open set (briefly, $\mathcal{PFN}\delta\mathcal{P}os$) if $S \subseteq \mathcal{PFN}int(\mathcal{PFN}\delta cl(S))$.

(iii) fuzzy nano $\delta\mathcal{S}$-open set (briefly, $\mathcal{PFN}\delta\mathcal{S}os$) if $S \subseteq \mathcal{PFN}cl(\mathcal{PFN}\delta int(S))$.

(iv) fuzzy nano $\delta\alpha$ or a-open set (briefly, $\mathcal{PFN}\delta\alpha os$ or $\mathcal{PFN}aos$) if $S \subseteq \mathcal{PFN}int(\mathcal{PFN}cl(\mathcal{PFN}\delta int(S)))$.

(v) fuzzy nano $\delta\beta$ or e^*-open set (briefly, $\mathcal{PFN}\delta\beta os$ or $\mathcal{PFN}e^*os$) if $S \subseteq \mathcal{PFN}cl(\mathcal{PFN}int(\mathcal{PFN}\delta cl(S)))$.

The complement of a $\mathcal{PFN}\delta os$ (resp. $\mathcal{PFN}\delta\mathcal{P}os$, $\mathcal{PFN}\delta\mathcal{S}os$, $\mathcal{PFN}\delta\alpha os$ & $\mathcal{PFN}\delta\beta os$) is called a Pythagorean fuzzy nano δ (resp. $\delta\mathcal{P}$, $\delta\mathcal{S}$, $\delta\alpha$ and $\delta\beta$) closed set (briefly, $\mathcal{PFN}\delta cs$ (resp. $\mathcal{PFN}\delta\mathcal{P}cs$, $\mathcal{PFN}\delta\mathcal{S}cs$, $\mathcal{PFN}\delta\alpha cs$ and $\mathcal{PFN}\delta\beta cs$)) in U.

The family of all $\mathcal{PFN}\delta os$ (resp. $\mathcal{PFN}\delta cs$, $\mathcal{PFN}\delta\mathcal{P}os$, $\mathcal{PFN}\delta\mathcal{P}cs$, $\mathcal{PFN}\delta\mathcal{S}os$, $\mathcal{PFN}\delta\mathcal{S}cs$, $\mathcal{PFN}\delta\alpha os$, $\mathcal{PFN}\delta\alpha cs$, $\mathcal{PFN}\delta\beta os$ and $\mathcal{PFN}\delta\beta cs$) of U is denoted by $\mathcal{PFN}\delta OS(U)$, (resp. $\mathcal{PFN}\delta CS(U)$, $\mathcal{PFN}\delta\mathcal{P}OS(U)$, $\mathcal{PFN}\delta\mathcal{P}CS(U)$, $\mathcal{PFN}\delta\mathcal{S}OS(U)$, $\mathcal{PFN}\delta\mathcal{S}CS(U)$, $\mathcal{PFN}\delta\alpha OS(U)$, $\mathcal{PFN}\delta\alpha CS(U)$, $\mathcal{PFN}\delta\beta OS(U)$ and $\mathcal{PFN}\delta\beta CS(U)$).

Definition 3.3. *Let $(U, \tau_{\mathcal{P}}(A))$ be a $\mathcal{PFN}ts$ with respect to A where A is a pfs of U. Let S be a pfs of U. Then Pythagorean fuzzy nano*

(i) δ pre (resp. δ semi, $\delta\alpha$ and $\delta\beta$) interior of S (briefly, $\mathcal{PFN}\delta\mathcal{P}int(S)$ (resp. $\mathcal{PFN}\delta\mathcal{S}int(S)$, $\mathcal{PFN}\delta\alpha int(S)$ and $\mathcal{PFN}\delta\beta int(S)$)) is defined by $\mathcal{PFN}\delta\mathcal{P}int$ (S) (resp. $\mathcal{PFN}\delta\mathcal{S}int(S), \mathcal{PFN}\delta\alpha int(S)$ and $\mathcal{PFN}\delta\beta int(S)) = \cup\{I : I \subseteq S$ & I is a $\mathcal{PFN}\delta\mathcal{P}o$ (resp. $\mathcal{PFN}\delta\mathcal{S}o, \mathcal{PFN}\delta\alpha o$ & $\mathcal{PFN}\delta\beta o$) set in $U\}$.

(ii) δ pre (resp. δ semi, $\delta\alpha$ and $\delta\beta$) closure of S (briefly, $\mathcal{PFN}\delta\mathcal{P}cl(S)$ (resp. $\mathcal{PFN}\delta\mathcal{S}cl(S)$, $\mathcal{PFN}\delta\alpha cl(S)$ and $\mathcal{PFN}\delta\beta cl(S)$)) is defined by $\mathcal{PFN}\delta\mathcal{P}cl(S)$ (resp. $\mathcal{PFN}\delta\mathcal{S}cl(S), \mathcal{PFN}\delta\alpha cl(S)$ and $\mathcal{PFN}\delta\beta cl(S)) = \cap\{A : S \subseteq A$ & A is a $\mathcal{PFN}\delta\mathcal{P}c$ (resp. $\mathcal{PFN}\delta\mathcal{S}c, \mathcal{PFN}\delta\alpha c$ & $\mathcal{PFN}\delta\beta c$) set in $U\}$.

Definition 3.4. *Let $(U_1, \tau_P(A_1))$ and $(U_2, \tau_P(A_2))$ be two $PF\mathcal{N}ts$'s. Then a function $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_p(A_2))$ is said to be a Pythagorean fuzzy nano δ (resp. δ pre, δ semi, $\delta\alpha$ and $\delta\beta$) continuous (briefly, $\mathcal{PFN}\delta Cts$ (resp. $\mathcal{PFN}\delta\mathcal{P}Cts$, $\mathcal{PFN}\delta\mathcal{S}Cts$, $\mathcal{PFN}\delta\alpha Cts$ and $\mathcal{PFN}\delta\beta Cts$)) function if $h_P^{-1}(G)$ is $\mathcal{PFN}\delta o$ (resp. $\mathcal{PFN}\delta\mathcal{P}o$, $\mathcal{PFN}\delta\mathcal{S}o$, $\mathcal{PFN}\delta\alpha o$ & $\mathcal{PFN}\delta\beta o$) set in U_1 for all $\mathcal{PFN}o$ set G in U_2.*

Lemma 3.1. *Let $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ be a function. Then the following statements hold.*

(i) If S and T are pfs's of U_1 such that $S \subseteq T$, then $h_P(S) \subseteq h_P(T)$.

(ii) If S and T are pfs's of U_2 such that $S \subseteq T$, then $h_P^{-1}(S) \subseteq h_P^{-1}(T)$.

Lemma 3.2. *Let $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ be a function. If S is a pfs of U_1 and T is a pfs of U_2. Then*

(i) $h_P(h_P^{-1}(S)) \subseteq S$

(ii) $h_P(h_P^{-1}(S)) = S \Leftrightarrow h_P$ is surjective.

(iii) $h_P^{-1}(h_P(S)) \supseteq S$

(iv) $h_P^{-1}(h_P(S)) = S$ whenever h_P is injective.

Theorem 3.1. *Let $(U_1, \tau_P(A_1))$ and $(U_2, \tau_P(A_2))$ be two $\mathcal{PFN}ts$'s and let $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$, then*

(i) Every $\mathcal{PFN}\delta Cts$ is a $\mathcal{PFN}Cts$.

(ii) Every $\mathcal{PFN}\delta Cts$ is a $\mathcal{PFN}\delta\mathcal{P}Cts$.

(iii) Every $\mathcal{PFN}\delta Cts$ is a $\mathcal{PFN}\delta\mathcal{S}Cts$.

(iv) Every $\mathcal{PFN}\delta\mathcal{S}Cts$ is a $\mathcal{PFN}\delta\beta Cts$.

(v) Every $\mathcal{PFN}\delta\mathcal{P}Cts$ is a $\mathcal{PFN}\delta\beta Cts$.

(vi) Every $\mathcal{PFN}\delta\alpha Cts$ is a $\mathcal{PFN}\delta\mathcal{P}Cts$.

(vii) Every $\mathcal{PFN}\delta\alpha Cts$ is a $\mathcal{PFN}\delta\mathcal{S}Cts$.

But not converse.

Proof. (i) Let $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ be a $\mathcal{PFN}\delta Cts$. Let S be a $\mathcal{PFN}o$ set in $(U_2, \tau_P(A_2))$. Then $h_P^{-1}(S)$ is $\mathcal{PFN}\delta o$ set in $(U_1, \tau_P(A_1))$. Since every $\mathcal{PFN}\delta o$ set is $\mathcal{PFN}os$, $h_P^{-1}(S)$ is $pf\mathcal{N}o$ set in $(U_1, \tau_P(A_1))$. Hence h_P is $\mathcal{PFN}Cts$ function.

(ii) Let $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ be a $\mathcal{PFN}\delta Cts$. Let S be a $\mathcal{PFN}o$ set in $(U_2, \tau_P(A_2))$. Then $h_P^{-1}(S)$ is $\mathcal{PFN}\delta o$ set in $(U_1, \tau_P(A_1))$. Since every $\mathcal{PFN}\delta o$ set is $\mathcal{PFN}\delta\mathcal{P}os$, $h_P^{-1}(S)$ is $pf\mathcal{N}\delta\mathcal{P}o$ set in $(U_1, \tau_P(A_1))$. Hence h_P is $\mathcal{PFN}\delta\mathcal{P}Cts$ function.

(iii) Let $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ be a $\mathcal{PFN}\delta Cts$. Let S be a $\mathcal{PFN}o$ set in $(U_2, \tau_P(A_2))$. Then $h_P^{-1}(S)$ is $\mathcal{PFN}\delta o$ set in $(U_1, \tau_P(A_1))$. Since every $\mathcal{PFN}\delta o$ set is $\mathcal{PFN}\delta\mathcal{S}os$, $h_P^{-1}(S)$ is $pf\mathcal{N}\delta\mathcal{S}o$ set in $(U_1, \tau_P(A_1))$. Hence h_P is $\mathcal{PFN}\delta\mathcal{S}Cts$ function.

(iv) Let $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ be a $\mathcal{PFN}\delta\mathcal{S}Cts$. Let S be a $\mathcal{PFN}o$ set in $(U_2, \tau_P(A_2))$. Then $h_P^{-1}(S)$ is $\mathcal{PFN}\delta\mathcal{S}o$ set in $(U_1, \tau_P(A_1))$. Since every $\mathcal{PFN}\delta\mathcal{S}o$ set is $\mathcal{PFN}\delta\beta os$, $h_P^{-1}(S)$ is $pf\mathcal{N}\delta\beta o$ set in $(U_1, \tau_P(A_1))$. Hence h_P is $\mathcal{PFN}\delta\beta Cts$ function.

(v) Let $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ be a $\mathcal{PFN}\delta\mathcal{P}Cts$. Let S be a $\mathcal{PFN}o$ set in $(U_2, \tau_P(A_2))$. Then $h_P^{-1}(S)$ is $\mathcal{PFN}\delta\mathcal{P}o$ set in $(U_1, \tau_P(A_1))$. Since every $\mathcal{PFN}\delta\mathcal{P}o$ set is $\mathcal{PFN}\delta\beta os$, $h_P^{-1}(S)$ is $pf\mathcal{N}\delta\beta o$ set in $(U_1, \tau_P(A_1))$. Hence h_P is $\mathcal{PFN}\delta\beta Cts$ function.

(vi) Let $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ be a $\mathcal{PFN}\delta\alpha Cts$. Let S be a $\mathcal{PFN}o$ set in $(U_2, \tau_P(A_2))$. Then $h_P^{-1}(S)$ is $\mathcal{PFN}\delta\alpha o$ set in $(U_1, \tau_P(A_1))$. Since every $\mathcal{PFN}\delta\alpha o$ set is $\mathcal{PFN}\delta\mathcal{P}os$, $h_P^{-1}(S)$ is $pf\mathcal{N}\delta\mathcal{P}o$ set in $(U_1, \tau_P(A_1))$. Hence h_P is $\mathcal{PFN}\delta\mathcal{P}Cts$ function.

(vii) Let $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ be a $\mathcal{PFN}\delta\alpha Cts$. Let S be a $\mathcal{PFN}o$ set in $(U_2, \tau_P(A_2))$. Then $h_P^{-1}(S)$ is $\mathcal{PFN}\delta\alpha o$ set in $(U_1, \tau_P(A_1))$. Since every $\mathcal{PFN}\delta\alpha o$ set is $\mathcal{PFN}\delta\mathcal{S}os$, $h_P^{-1}(S)$ is $pf\mathcal{N}\delta\mathcal{S}o$ set in $(U_1, \tau_P(A_1))$. Hence h_P is $\mathcal{PFN}\delta\mathcal{S}Cts$ function.

Remark 3.1. *The following Figure shows the relations among the different types of Pythagorean fuzzy δ continuous mappings that were studied in this section.*

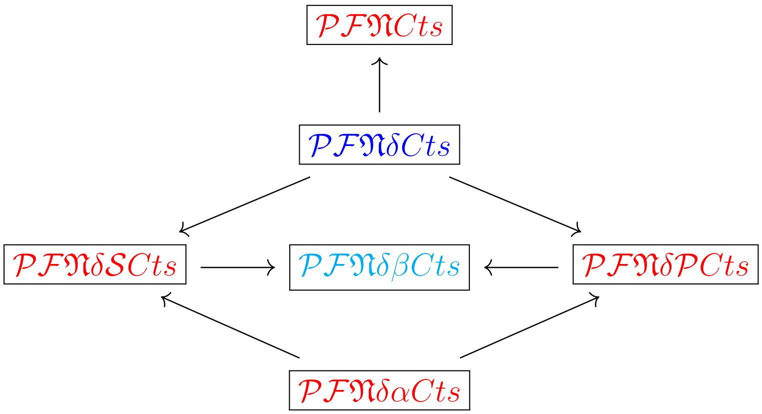

Figure : $\mathcal{PFN}\delta Cts$ mappings in $\mathcal{PFN}ts$

Example 3.1. Assume $U_1 = U_2 = U = \{s_1, s_2, s_3, s_4\}$ be the universe set and the equivalence relation is $U/R = \{\{s_1, s_4\}, \{s_2\}, \{s_3\}\}$.
Let $A = \left\{\left\langle \frac{s_1}{0.3,0.1} \right\rangle, \left\langle \frac{s_2}{0.1,0.5} \right\rangle, \left\langle \frac{s_3}{0.2,0.45} \right\rangle, \left\langle \frac{s_4}{0.4,0.25} \right\rangle\right\}$ be a Pythagorean fuzzy subset of U.

$$\underline{\mathcal{PFN}}(A) = \left\{\left\langle \frac{s_1, s_4}{0.3, 0.25} \right\rangle, \left\langle \frac{s_2}{0.1, 0.5} \right\rangle, \left\langle \frac{s_3}{0.2, 0.45} \right\rangle\right\},$$
$$\overline{\mathcal{PFN}}(A) = \left\{\left\langle \frac{s_1, s_4}{0.4, 0.1} \right\rangle, \left\langle \frac{s_2}{0.1, 0.5} \right\rangle, \left\langle \frac{s_3}{0.2, 0.45} \right\rangle\right\},$$
$$B_{\mathcal{PFN}}(A) = \left\{\left\langle \frac{s_1, s_4}{0.25, 0.3} \right\rangle, \left\langle \frac{s_2}{0.1, 0.5} \right\rangle, \left\langle \frac{s_3}{0.2, 0.45} \right\rangle\right\}.$$

Now $\tau_P(A_1) = \tau_P(A_2) = \tau_P(A) = \{0_{\mathcal{P}}, 1_{\mathcal{P}}, \underline{\mathcal{PFN}}(A), \overline{\mathcal{PFN}}(A), B_{\mathcal{PFN}}(A)\}$. Let $h_P : (U, \tau_P(A_1)) \to (U, \tau_P(A_2))$ be an identity function, Then h_P is $\mathcal{PFN}Cts$ (resp. $\mathcal{PFN}\delta\mathcal{P}Cts$, $\mathcal{PFN}\delta\beta Cts$ and $\mathcal{PFN}\delta\mathcal{P}Cts$) but not $\mathcal{PFN}\delta Cts$ (resp. $\mathcal{PFN}\delta Cts$, $\mathcal{PFN}\delta\mathcal{S}Cts$ and $\mathcal{PFN}\delta\alpha Cts$). Since, $\overline{\mathcal{PFN}}(A)$ is a $\mathcal{PFN}o$ set in U_2 but h_P^{-1} $(\overline{\mathcal{PFN}}(A)) = \overline{\mathcal{PFN}}(A)$ is not $\mathcal{PFN}\delta o$ (resp. $\mathcal{PFN}\delta o$, $\mathcal{PFN}\delta\mathcal{S}o$ and $\mathcal{PFN}\delta\alpha o$) set in U_1.

Example 3.2. Let $U_1 = \{s_1, s_2, s_3, s_4\}$, $U_2 = \{t_1, t_2, t_3, t_4\}$ are the universe sets and the equivalence relations are $U_1/R = \{\{s_1, s_4\}, \{s_2\}, \{s_3\}\}$ and $U_2/R = \{\{t_1, t_4\}, \{t_2\}, \{t_3\}\}$. Let $A_1 = \left\{\left\langle \frac{s_1}{0.3,0.7} \right\rangle, \left\langle \frac{s_2}{0.1,0.6} \right\rangle, \left\langle \frac{s_3}{0.4,0.4} \right\rangle, \left\langle \frac{s_4}{0.4,0.6} \right\rangle\right\}$ and

$A_2 = \left\{\left\langle\frac{t_1}{0.6,0.4}\right\rangle, \left\langle\frac{t_2}{0.6,0.2}\right\rangle, \left\langle\frac{t_3}{0.6,0.4}\right\rangle, \left\langle\frac{t_4}{0.6,0.4}\right\rangle\right\}$ be a Pythagorean fuzzy subsets of U_1 and U_2 respectively.

$$\underline{\mathcal{PF}\mathfrak{N}}(A_1) = \left\{\left\langle\frac{s_1, s_4}{0.3, 0.7}\right\rangle, \left\langle\frac{s_2}{0.1, 0.6}\right\rangle, \left\langle\frac{s_3}{0.4, 0.4}\right\rangle\right\},$$
$$\overline{\mathcal{PF}\mathfrak{N}}(A_1) = \left\{\left\langle\frac{s_1, s_4}{0.4, 0.6}\right\rangle, \left\langle\frac{s_2}{0.1, 0.6}\right\rangle, \left\langle\frac{s_3}{0.4, 0.4}\right\rangle\right\},$$
$$B_{\mathcal{PF}\mathfrak{N}}(A_1) = \left\{\left\langle\frac{s_1, s_4}{0.4, 0.6}\right\rangle, \left\langle\frac{s_2}{0.1, 0.6}\right\rangle, \left\langle\frac{s_3}{0.4, 0.4}\right\rangle\right\},$$
$$\underline{\mathcal{PF}\mathfrak{N}}(A_2) = \left\{\left\langle\frac{t_1, t_4}{0.6, 0.4}\right\rangle, \left\langle\frac{t_2}{0.6, 0.2}\right\rangle, \left\langle\frac{t_3}{0.6, 0.4}\right\rangle\right\},$$
$$\overline{\mathcal{PF}\mathfrak{N}}(A_2) = \left\{\left\langle\frac{t_1, t_4}{0.6, 0.4}\right\rangle, \left\langle\frac{t_2}{0.6, 0.2}\right\rangle, \left\langle\frac{t_3}{0.6, 0.4}\right\rangle\right\},$$
$$B_{\mathcal{PF}\mathfrak{N}}(A_2) = \left\{\left\langle\frac{t_1, t_4}{0.4, 0.6}\right\rangle, \left\langle\frac{t_2}{0.2, 0.6}\right\rangle, \left\langle\frac{t_3}{0.4, 0.6}\right\rangle\right\}.$$

Now $\tau_P(A_1) = \{0_{\mathcal{P}}, 1_{\mathcal{P}}, \underline{\mathcal{PF}\mathfrak{N}}(A_1), \overline{\mathcal{PF}\mathfrak{N}}(A_1) = B_{\mathcal{PF}\mathfrak{N}}(A_1)\}$, $\tau_P(A_2) = \{0_{\mathcal{P}}, 1_{\mathcal{P}}, \underline{\mathcal{PF}\mathfrak{N}}(A_2) = \overline{\mathcal{PF}\mathfrak{N}}(A_2), B_{\mathcal{PF}\mathfrak{N}}(A_2)\}$. Let $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ be an identity function, then h_P is $\mathcal{PF}\mathfrak{N}\delta\mathcal{S}Cts$ but not $\mathcal{PF}\mathfrak{N}\delta Cts$ (resp. $\mathcal{PF}\mathfrak{N}\delta\alpha Cts$). Since, $B_{\mathcal{PF}\mathfrak{N}}(A_2)$ is a $\mathcal{PF}\mathfrak{N}o$ set in U_2 but $h_P^{-1}(B_{\mathcal{PF}\mathfrak{N}}(A_2)) = B_{\mathcal{PF}\mathfrak{N}}(A_2)$ is not $\mathcal{PF}\mathfrak{N}\delta o$ (resp. $\mathcal{PF}\mathcal{N}\delta\alpha o$) set in U_1.

Example 3.3. Let $U_1 = \{s_1, s_2, s_3, s_4\}$, $U_2 = \{t_1, t_2, t_3, t_4\}$ are the universe sets and the equivalence relations are $U_1/R = \{\{s_1, s_4\}, \{s_2\}, \{s_3\}\}$ and $U_2/R = \{\{t_1, t_4\}, \{t_2\}, \{t_3\}\}$. Let $A_1 = \left\{\left\langle\frac{s_1}{0.3,0.1}\right\rangle, \left\langle\frac{s_2}{0.1,0.5}\right\rangle, \left\langle\frac{s_3}{0.2,0.45}\right\rangle, \left\langle\frac{s_4}{0.4,0.25}\right\rangle\right\}$ and $A_2 = \left\{\left\langle\frac{t_1}{0.4,0.3}\right\rangle, \left\langle\frac{t_2}{0.4,0.2}\right\rangle, \left\langle\frac{t_3}{0.5,0.3}\right\rangle, \left\langle\frac{t_4}{0.5,0.2}\right\rangle\right\}$ be a Pythagorean fuzzy subsets of U.

$$\underline{\mathcal{PF}\mathfrak{N}}(A_1) = \left\{\left\langle\frac{s_1, s_4}{0.3, 0.25}\right\rangle, \left\langle\frac{s_2}{0.1, 0.5}\right\rangle, \left\langle\frac{s_3}{0.2, 0.45}\right\rangle\right\},$$
$$\overline{\mathcal{PF}\mathfrak{N}}(A_1) = \left\{\left\langle\frac{s_1, s_4}{0.4, 0.1}\right\rangle, \left\langle\frac{s_2}{0.1, 0.5}\right\rangle, \left\langle\frac{s_3}{0.2, 0.45}\right\rangle\right\},$$
$$B_{\mathcal{PF}\mathfrak{N}}(A_1) = \left\{\left\langle\frac{s_1, s_4}{0.25, 0.3}\right\rangle, \left\langle\frac{s_2}{0.1, 0.5}\right\rangle, \left\langle\frac{s_3}{0.2, 0.45}\right\rangle\right\},$$
$$\underline{\mathcal{PF}\mathfrak{N}}(A_2) = \left\{\left\langle\frac{t_1, t_4}{0.4, 0.3}\right\rangle, \left\langle\frac{t_2}{0.4, 0.2}\right\rangle, \left\langle\frac{t_3}{0.5, 0.3}\right\rangle\right\},$$

$$\overline{\mathcal{PFN}}(A_2) = \left\{ \left\langle \frac{t_1, t_4}{0.5, 0.2} \right\rangle, \left\langle \frac{t_2}{0.4, 0.2} \right\rangle, \left\langle \frac{t_3}{0.5, 0.3} \right\rangle \right\},$$

$$B_{\mathcal{PFN}}(A_2) = \left\{ \left\langle \frac{t_1, t_4}{0.3, 0.4} \right\rangle, \left\langle \frac{t_2}{0.2, 0.4} \right\rangle, \left\langle \frac{t_3}{0.3, 0.5} \right\rangle \right\}.$$

Now $\tau_P(A_1) = \{0_{\mathcal{P}}, 1_{\mathcal{P}}, \underline{\mathcal{PFN}}(A_1), \overline{\mathcal{PFN}}(A_1), B_{\mathcal{PFN}}(A_1)\}$, $\tau_P(A_2) = \{0_{\mathcal{P}}, 1_{\mathcal{P}}, \underline{\mathcal{PFN}}(A_2), \overline{\mathcal{PFN}}(A_2), B_{\mathcal{PFN}}(A_2)\}$. Let $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ be an identity function, Then h_P is $\mathcal{PFN}\delta\beta Cts$ but not $\mathcal{PFN}\delta\mathcal{P}Cts$. Since, $B_{\mathcal{PFN}}(A_2)$ is a $\mathcal{PFN}o$ set in U_2 but $h_P^{-1}(B_{\mathcal{PFN}}(A_2)) = B_{\mathcal{PFN}}(A_2)$ is not $\mathcal{PFN}\delta\mathcal{P}o$ set in U_1.

Theorem 3.2. *Let $(U_1, \tau_P(A_1))$ & $(U_2, \tau_P(A_2))$ be a $\mathcal{PFN}ts$'s. A mapping $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ satisfies the following conditions are equivalent.*

(i) h_P is $\mathcal{PFN}\delta\beta Cts$;

(ii) The inverse $h_P^{-1}(K)$ of all $\mathcal{PFN}cs$ set K in U_2 is $\mathcal{PFN}\delta\beta cs$ in U_1.

Proof. The proof is directly, from $h_P^{-1}(\overline{K}) = \overline{h_P^{-1}(K)}$ for all $\mathcal{PFN}cs$ K of U_2.

Theorem 3.3. *Let $(U_1, \tau_P(A_1))$ & $(U_2, \tau_P(A_2))$ be a $\mathcal{PFN}ts$'s. A mapping $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ satisfies the following conditions are hold.*

(i) $h_P(\mathcal{PFN}\delta\beta cl(L)) \subseteq \mathcal{PFN}\delta cl(h_P(L))$, for all $\mathcal{PFN}cs$ L in U_1.

(ii) $\mathcal{PFN}\delta\beta cl(h_P^{-1}(K)) \subseteq h_P^{-1}(\mathcal{PFN}\delta cl(K))$, for all $\mathcal{PFN}cs$ K in U_2.

Proof. (i) Since $\mathcal{PFN}\delta cl(h_P(L))$ is a $\mathcal{PFN}\delta cs$ in U_2 and h_P is $\mathcal{PFN}\delta\beta Cts$, then $h_P^{-1}(\mathcal{PFN}\delta cl(\ h_P(L)))$ is $\mathcal{PFN}\delta\beta cs$ in U_1. Now, since $L \subseteq h_P^{-1}(\mathcal{PFN}\delta cl(h_P(L)))$, $\mathcal{PFN}\delta\beta cl(L) \subseteq h_P^{-1}(\mathcal{PFN}\delta cl(h_P(L)))$. Therefore, $h_P(\mathcal{PFN}\delta\beta cl(L)) \subseteq \mathcal{PFN}\delta cl(h_P(L))$.
(ii) By replacing L with K in (i), we obtain $h_P(\mathcal{PFN}\delta\beta cl(h_P^{-1}(K))) \subseteq \mathcal{PFN}\delta cl(h_P(\ h_P^{-1}(K))) \subseteq \mathcal{PFN}\delta cl(K)$. Hence, $\mathcal{PFN}\delta\beta cl(h_P^{-1}(K)) \subseteq h_P^{-1}(\mathcal{PFN}\delta cl(K))$.

Remark 3.2. *Let $(U_1, \tau_P(A_1))$ & $(U_2, \tau_P(A_2))$ be a $\mathcal{PFN}ts$'s. Let $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ be a mapping. If h_P is $\mathcal{PFN}\delta\beta Cts$, then*

(i) $h_P(\mathcal{PFN}\delta\beta cl(L))$ is not necessarily equal to $\mathcal{PFN}\delta cl(h_P(L))$ where $L \in U_1$.

(ii) $\mathcal{PFN}\delta\beta cl(h_P^{-1}(K))$ is not necessarily equal to $h_P^{-1}(\mathcal{PFN}\delta cl(K))$ where $K \in U_2$.

Example 3.4. Assume $U_1 = U_2 = U = \{s_1, s_2, s_3, s_4\}$ be the universe set and the equivalence relation is $U/R = \{\{s_1, s_4\}, \{s_2\}, \{s_3\}\}$.

Let $A = \left\{ \left\langle \frac{s_1}{0.3,0.1} \right\rangle, \left\langle \frac{s_2}{0.1,0.5} \right\rangle, \left\langle \frac{s_3}{0.2,0.45} \right\rangle, \left\langle \frac{s_4}{0.4,0.25} \right\rangle \right\}$ be a Pythagorean fuzzy subset of U.

$$\underline{\mathcal{PFN}}(A) = \left\{ \left\langle \frac{s_1, s_4}{0.3, 0.25} \right\rangle, \left\langle \frac{s_2}{0.1, 0.5} \right\rangle, \left\langle \frac{s_3}{0.2, 0.45} \right\rangle \right\},$$
$$\overline{\mathcal{PFN}}(A) = \left\{ \left\langle \frac{s_1, s_4}{0.4, 0.1} \right\rangle, \left\langle \frac{s_2}{0.1, 0.5} \right\rangle, \left\langle \frac{s_3}{0.2, 0.45} \right\rangle \right\},$$
$$B_{\mathcal{PFN}}(A) = \left\{ \left\langle \frac{s_1, s_4}{0.25, 0.3} \right\rangle, \left\langle \frac{s_2}{0.1, 0.5} \right\rangle, \left\langle \frac{s_3}{0.2, 0.45} \right\rangle \right\}.$$

Now $\tau_P(A_1) = \tau_P(A_2) = \tau_P(A) = \{0_{\mathcal{P}}, 1_{\mathcal{P}}, \underline{\mathcal{PFN}}(A), \overline{\mathcal{PFN}}(A), B_{\mathcal{PFN}}(A)\}$ is a $\mathcal{PFN}ts$. Let $h_P : (U, \tau_P(A_1)) \to (U, \tau_P(A_2))$ be an identity function, then h_P is $\mathcal{PFN}\delta\beta Cts$.

(i) $h_P(\mathcal{PFN}\delta\beta cl(\underline{\mathcal{PFN}}(A))) = B_{\mathcal{PFN}}(A)$. But $\mathcal{PFN}\delta cl(h_P(\underline{\mathcal{PFN}}(A))) = B_{\mathcal{PFN}}(A)^c$. Thus $h_P(\mathcal{PFN}\delta\beta cl(\underline{\mathcal{PFN}}(A))) \neq \mathcal{PFN}\delta cl(h_P(\underline{\mathcal{PFN}}(A)))$.

(ii) $\mathcal{PFN}\delta\beta cl(h_P^{-1}(\underline{\mathcal{PFN}}(A))) = B_{\mathcal{PFN}}(A)$. But $h_P^{-1}(\mathcal{PFN}\delta cl(\underline{\mathcal{PFN}}(A))) = B_{\mathcal{PFN}}(A)^c$. Thus $\mathcal{PFN}\delta\beta cl(h_P^{-1}(\underline{\mathcal{PFN}}(A))) \neq h_P^{-1}(\mathcal{PFN}\delta cl(\underline{\mathcal{PFN}}(A)))$.

Theorem 3.4. *Let $(U_1, \tau_P(A_1))$ & $(U_2, \tau_P(A_2))$ be a $\mathcal{PFN}ts$'s. Let $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ be a mapping. If h_P is $\mathcal{PFN}\delta\beta Cts$, then $h_P^{-1}(\mathcal{PFN}\delta int(L)) \subseteq \mathcal{PFN}\delta\beta int(h_P^{-1}(L))$, for all pfs L in U_2.*

Proof. If h_P is $\mathcal{PFN}\delta\beta Cts$ and $L \subseteq U_2$. $\mathcal{PFN}\delta int(L)$ is $\mathcal{PFN}\delta os$ in U_2 and hence, $h_P^{-1}(\mathcal{PFN}\delta\ int(L))$ is $\mathcal{PFN}\delta\beta os$ in U_1. Therefore $\mathcal{PFN}\delta\beta int(h_P^{-1}(\mathcal{PF}\mathcal{N}\delta int(L))) = h_P^{-1}(\mathcal{PFN}\delta int(L))$. Also, $\mathcal{PFN}\delta int(L) \subseteq L$, implies that $h_P^{-1}(\mathcal{PFN}\delta int(L)) \subseteq h_P^{-1}(L)$. Therefore $\mathcal{PFN}\delta\beta int(h_P^{-1}(\mathcal{PFN}\delta int(L))) \subseteq \mathcal{PFN}\delta\beta int(h_P^{-1}(L))$. That is $h_P^{-1}(\mathcal{PFN}\delta int(L)) \subseteq \mathcal{PFN}\delta\beta int(h_P^{-1}(L))$.

Conversely, let $h_P^{-1}(\mathcal{PFN}\delta int(L)) \subseteq \mathcal{PFN}\delta\beta int(h_P^{-1}(L))$ for all subset L of U_2. If L is $\mathcal{PFN}\delta o$ in U_2, then $\mathcal{PFN}\delta int(L) = L$. By assumption, $h_P^{-1}(\mathcal{PFN}\delta int(L)) \subseteq \mathcal{PFN}\delta\beta int(h_P^{-1}(L))$. Thus $h_P^{-1}(L) \subseteq \mathcal{PFN}\delta\beta int(h_P^{-1}(L))$. But $\mathcal{PFN}\delta\beta int\ (h_P^{-1}(L)) \subseteq h_P^{-1}(L)$. Therefore $\mathcal{PFN}\delta\beta int(h_P^{-1}(L)) = h_P^{-1}(L)$. That is, $h_P^{-1}(L)$ is $\mathcal{PFN}\delta\beta o$ in U_1, for all $\mathcal{PFN}\delta os$ L in U_2. Therefore h_P is $\mathcal{PFN}\delta\beta Cts$ in U_1.

Remark 3.3. *Let $(U_1, \tau_P(A_1))$ & $(U_2, \tau_P(A_2))$ be a $\mathcal{PFN}ts$'s. Let $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ be a mapping. If h_P is $\mathcal{PFN}\delta\beta Cts$, then $\mathcal{PFN}\delta\beta int(h_P^{-1}(K))$ is not necessarily equal to $h_P^{-1}(\mathcal{PFN}\delta int(K))$ where $K \in U_2$.*

Example 3.5. In Example 3.4, h_P is a $\mathcal{PFN}\delta\beta Cts$.
Then $\mathcal{PFN}\delta\beta int(h_P^{-1}(\underline{\mathcal{PFN}}(A))) = B_{\mathcal{PFN}}(A)^c$. But $h_P^{-1}(\mathcal{PFN}\delta int(\underline{\mathcal{PFN}}(A))) =$

$B_{\mathcal{PFN}}(A)$. Thus $\mathcal{PFN}\delta\beta int(h_P^{-1}(K)) \neq h_P^{-1}(\mathcal{PFN}\delta int(K))$.

Remark 3.4. *Theorems 3.2, 3.3, 3.4 and Remarks 3.2, 3.3 are true for $\mathcal{PFN}\delta\mathcal{P}os$, $\mathcal{PFN}\delta\mathcal{S}os$ and $\mathcal{PFN}\delta\alpha os$.*

4. Pythagorean fuzzy nano δ (resp. δ pre, δ semi, $\delta\alpha$ and $\delta\beta$)-irresolute maps

In this section, we introduce the concept of Pythagorean fuzzy nano irresoluteness called Pythagorean fuzzy nano (resp. δ, $\delta\mathcal{P}$, $\delta\mathcal{S}$, $\delta\alpha$ and $\delta\beta$)-irresolute maps by using $\mathcal{PFNS}os$ (resp. $\mathcal{PFN}\delta os$, $\mathcal{PFN}\delta\mathcal{P}os$, $\mathcal{PFN}\delta\mathcal{S}os$, $\mathcal{PFN}\delta\alpha os$ and $\mathcal{PFN}\delta\beta os$)'s and study some of their basic properties. This definition enables us to obtain conditions under which maps and inverse maps preserve respective open sets.

Definition 4.1. *A map $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ is said to be Pythagorean fuzzy nano (resp. δ, $\delta\mathcal{P}$, $\delta\mathcal{S}$, $\delta\alpha$ and $\delta\beta$)-irresolute (in short, $\mathcal{PFN}Irr$ (resp. $\mathcal{PFN}\delta Irr$, $\mathcal{PFN}\delta\mathcal{P}Irr$, $\mathcal{PFN}\delta\mathcal{S}Irr$, $\mathcal{PFN}\delta\alpha Irr$ and $\mathcal{PFN}\delta\beta Irr$)) map if $h_P^{-1}(K)$ is a $\mathcal{PFNS}os$ (resp. $\mathcal{PFN}\delta os$, $\mathcal{PFN}\delta\mathcal{P}os$, $\mathcal{PFN}\delta\mathcal{S}os$, $\mathcal{PFN}\delta\alpha os$ and $\mathcal{PFN}\delta\beta os$) in $(U_1, \tau_P(A_1))$ for each $\mathcal{PFNS}os$ (resp. $\mathcal{PFN}\delta os$,$\mathcal{PFN}\delta\mathcal{P}os$, $\mathcal{PFN}\delta\mathcal{S}os$, $\mathcal{PFN}\delta$ αos and $\mathcal{PFN}\delta\beta os$) K of $(U_2, \tau_P(A_2))$.*

Theorem 4.1. *Let $(U_1, \tau_P(A_1))$ & $(U_2, \tau_P(A_2))$ be a $\mathcal{PFN}ts$'s. Let $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ be a mapping. Then the following statements are hold for $\mathcal{PFN}ts$, but not conversely.*

(i) Every $\mathcal{PFN}Irr$ map is a $\mathcal{PFNS}Cts$.

(ii) Every $\mathcal{PFN}\delta\mathcal{S}Irr$ map is a $\mathcal{PFN}\delta\mathcal{S}Cts$.

(iii) Every $\mathcal{PFN}\delta\mathcal{P}Irr$ map is a $\mathcal{PFN}\delta\mathcal{P}Cts$.

(iv) Every $\mathcal{PFN}\delta\alpha Irr$ map is a $\mathcal{PFN}\delta\alpha Cts$.

(v) Every $\mathcal{PFN}\delta\beta Irr$ map is a $\mathcal{PFN}\delta\beta Cts$.

Proof. (i) Consider a $\mathcal{PFN}Irr$ map h_P and a $\mathcal{PFN}os$ K in U_2. As each $\mathcal{PFN}os$ is a $\mathcal{PFNS}os$, K is a $\mathcal{PFNS}os$ in U_2. By presumption, $h_P^{-1}(K)$ is a $\mathcal{PFNS}os$ in U_1. Thus h_P is a $\mathcal{PFNS}Cts$ map.
(ii) Consider a $\mathcal{PFN}\delta\mathcal{S}Irr$ map h_P and a $\mathcal{PFN}\delta os$ K in U_2. As each $\mathcal{PFN}\delta os$ is a $\mathcal{PFN}os$ and $\mathcal{PFN}\delta\mathcal{S}os$, K is a $\mathcal{PFN}\delta os$ and $\mathcal{PFN}\delta\mathcal{S}os$ in U_2. By presumption, $h_P^{-1}(K)$ is a $\mathcal{PFN}\delta\mathcal{S}os$ in U_1. Thus h_P is a $\mathcal{PFN}\delta\mathcal{S}Cts$ map.
(iii) Consider a $\mathcal{PFN}\delta\mathcal{P}Irr$ map h_P and a $\mathcal{PFN}\delta os$ K in U_2. As each $\mathcal{PFN}\delta os$ is a $\mathcal{PFN}os$ and $\mathcal{PFN}\delta\mathcal{P}os$, K is a $\mathcal{PFN}\delta os$ and $\mathcal{PFN}\delta\mathcal{P}os$ in U_2. By presumption,

$h_P^{-1}(K)$ is a $\mathcal{PF}\mathfrak{N}\delta\mathcal{P}os$ in U_1. Thus h_P is a $\mathcal{PF}\mathfrak{N}\delta\mathcal{P}Cts$ map.
(iv) Consider a $\mathcal{PF}\mathfrak{N}\delta\alpha Irr$ map h_P and a $\mathcal{PF}\mathfrak{N}\delta os$ K in U_2. As each $\mathcal{PF}\mathfrak{N}\delta os$ is a $\mathcal{PF}\mathfrak{N}os$ and $\mathcal{PF}\mathfrak{N}\delta\alpha os$, K is a $\mathcal{PF}\mathfrak{N}\delta os$ and $\mathcal{PF}\mathfrak{N}\delta\alpha os$ in U_2. By presumption, $h_P^{-1}(K)$ is a $\mathcal{PF}\mathfrak{N}\delta\alpha os$ in U_1. Thus h_P is a $\mathcal{PF}\mathfrak{N}\delta\alpha Cts$ map.
(v) Consider a $\mathcal{PF}\mathfrak{N}\delta\beta Irr$ map h_P and a $\mathcal{PF}\mathfrak{N}\delta os$ K in U_2. As each $\mathcal{PF}\mathfrak{N}\delta os$ is a $\mathcal{PF}\mathfrak{N}os$ and $\mathcal{PF}\mathfrak{N}\delta\beta os$, K is a $\mathcal{PF}\mathfrak{N}\delta os$ and $\mathcal{PF}\mathfrak{N}\delta\beta os$ in U_2. By presumption, $h_P^{-1}(K)$ is a $\mathcal{PF}\mathfrak{N}\delta\beta os$ in U_1. Thus h_P is a $\mathcal{PF}\mathfrak{N}\delta\beta Cts$ map.

Example 4.1. Let $U_1 = \{s_1, s_2, s_3, s_4\}$, $U_2 = \{t_1, t_2, t_3, t_4\}$ are the universe sets and the equivalence relations are $U_1/R = \{\{s_1, s_4\}, \{s_2, s_3\}\}$ and $U_2/R = \{\{t_1, t_4\}, \{t_2, t_3\}\}$. Let $A_1 = \left\{\left\langle\frac{s_1}{0.1,0.8}\right\rangle, \left\langle\frac{s_2}{0.3,0.7}\right\rangle, \left\langle\frac{s_3}{0.2,0.9}\right\rangle, \left\langle\frac{s_4}{0.4,0.6}\right\rangle\right\}$ and $A_2 = \left\{\left\langle\frac{t_1}{0.4,0.7}\right\rangle, \left\langle\frac{t_2}{0.5,0.7}\right\rangle, \left\langle\frac{t_3}{0.4,0.6}\right\rangle, \left\langle\frac{t_4}{0.3,0.7}\right\rangle\right\}$ be a Pythagorean fuzzy subsets of U_1 and U_2 respectively.

$$\underline{\mathcal{PF}\mathfrak{N}}(A_1) = \left\{\left\langle\frac{s_1, s_4}{0.1, 0.9}\right\rangle, \left\langle\frac{s_2, s_3}{0.3, 0.7}\right\rangle\right\},$$
$$\overline{\mathcal{PF}\mathfrak{N}}(A_1) = \left\{\left\langle\frac{s_1, s_4}{0.2, 0.8}\right\rangle, \left\langle\frac{s_2, s_3}{0.4, 0.6}\right\rangle\right\},$$
$$B_{\mathcal{PF}\mathfrak{N}}(A_1) = \left\{\left\langle\frac{s_1, s_4}{0.2, 0.8}\right\rangle, \left\langle\frac{s_2, s_3}{0.4, 0.6}\right\rangle\right\},$$
$$\underline{\mathcal{PF}\mathfrak{N}}(A_2) = \left\{\left\langle\frac{t_1, t_4}{0.4, 0.7}\right\rangle, \left\langle\frac{t_2, t_3}{0.3, 0.7}\right\rangle\right\},$$
$$\overline{\mathcal{PF}\mathfrak{N}}(A_2) = \left\{\left\langle\frac{t_1, t_4}{0.4, 0.6}\right\rangle, \left\langle\frac{t_2, t_3}{0.5, 0.7}\right\rangle\right\},$$
$$B_{\mathcal{PF}\mathfrak{N}}(A_2) = \left\{\left\langle\frac{t_1, t_4}{0.4, 0.6}\right\rangle, \left\langle\frac{t_2, t_3}{0.5, 0.7}\right\rangle\right\}.$$

Here $\tau_p(A_1) = \{0_P, 1_P, \underline{\mathcal{PF}\mathfrak{N}}(A_1), \overline{\mathcal{PF}\mathfrak{N}}(A_1) = B_{\mathcal{PF}\mathfrak{N}}(A_1)\}$ and $\tau_p(A_2) = \{0_P, 1_P, \underline{\mathcal{PF}\mathfrak{N}}(A_2), \overline{\mathcal{PF}\mathfrak{N}}(A_2) = B_{\mathcal{PF}\mathfrak{N}}(A_2)\}$ are the $\mathcal{PF}\mathfrak{N}ts's$ on U_1 and U_2 respectively.

Let $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ be an identity function, then h_P is $\mathcal{PF}\mathfrak{N}\mathcal{S}$ Cts but not $\mathcal{PF}\mathfrak{N}Irr$, because the set $B_{\mathcal{PF}\mathfrak{N}}(A_2)^c$ is a $\mathcal{PF}\mathfrak{N}\mathcal{S}os$ in U_2 but h_P^{-1} $(B_{\mathcal{PF}\mathfrak{N}}(A_2)^c) = B_{\mathcal{PF}\mathfrak{N}}(A_2)^c$ is not $\mathcal{PF}\mathfrak{N}\mathcal{S}os$ in U_1.

Example 4.2. Let $U_1 = \{s_1, s_2, s_3, s_4\}$, $U_2 = \{t_1, t_2, t_3, t_4\}$ are the universe sets and the equivalence relations are $U_1/R = \{\{s_1, s_4\}, \{s_2\}, \{s_3\}\}$ and $U_2/R = \{\{t_1, t_4\}, \{t_2\}, \{t_3\}\}$. Let $A_1 = \left\{\left\langle\frac{s_1}{0.4,0.7}\right\rangle, \left\langle\frac{s_2}{0.2,0.6}\right\rangle, \left\langle\frac{s_3}{0.5,0.5}\right\rangle, \left\langle\frac{s_4}{0.3,0.8}\right\rangle\right\}$ and $A_2 = \left\{\left\langle\frac{t_1}{0.7,0.6}\right\rangle, \left\langle\frac{t_2}{0.3,0.7}\right\rangle, \left\langle\frac{t_3}{0.5,0.5}\right\rangle, \left\langle\frac{t_4}{0.7,0.6}\right\rangle\right\}$ be a Pythagorean fuzzy subsets of U_1 and U_2 respectively.

$$\underline{\mathcal{PF}\mathfrak{N}}(A_1) = \left\{ \left\langle \frac{s_1, s_4}{0.3, 0.8} \right\rangle, \left\langle \frac{s_2}{0.2, 0.6} \right\rangle, \left\langle \frac{s_3}{0.5, 0.5} \right\rangle \right\},$$
$$\overline{\mathcal{PF}\mathfrak{N}}(A_1) = \left\{ \left\langle \frac{s_1, s_4}{0.4, 0.7} \right\rangle, \left\langle \frac{s_2}{0.2, 0.6} \right\rangle, \left\langle \frac{s_3}{0.5, 0.5} \right\rangle \right\},$$
$$B_{\mathcal{PF}\mathfrak{N}}(A_1) = \left\{ \left\langle \frac{s_1, s_4}{0.4, 0.7} \right\rangle, \left\langle \frac{s_2}{0.2, 0.6} \right\rangle, \left\langle \frac{s_3}{0.5, 0.5} \right\rangle \right\},$$
$$\underline{\mathcal{PF}\mathfrak{N}}(A_2) = \left\{ \left\langle \frac{t_1, t_4}{0.7, 0.6} \right\rangle, \left\langle \frac{t_2}{0.3, 0.7} \right\rangle, \left\langle \frac{t_3}{0.5, 0.5} \right\rangle \right\},$$
$$\overline{\mathcal{PF}\mathfrak{N}}(A_2) = \left\{ \left\langle \frac{t_1, t_4}{0.7, 0.6} \right\rangle, \left\langle \frac{t_2}{0.3, 0.7} \right\rangle, \left\langle \frac{t_3}{0.5, 0.5} \right\rangle \right\},$$
$$B_{\mathcal{PF}\mathfrak{N}}(A_2) = \left\{ \left\langle \frac{t_1, t_4}{0.6, 0.7} \right\rangle, \left\langle \frac{t_2}{0.3, 0.7} \right\rangle, \left\langle \frac{t_3}{0.5, 0.5} \right\rangle \right\}.$$

Here $\tau_p(A_1) = \{0_P, 1_P, \underline{\mathcal{PF}\mathfrak{N}}(A_1), \overline{\mathcal{PF}\mathfrak{N}}(A_1) = B_{\mathcal{PF}\mathfrak{N}}(A_1)\}$ and $\tau_p(A_2) = \{0_P, 1_P, \underline{\mathcal{PF}\mathfrak{N}}(A_2) = \overline{\mathcal{PF}\mathfrak{N}}(A_2), B_{\mathcal{PF}\mathfrak{N}}(A_2)\}$ are the $\mathcal{PF}\mathfrak{N}ts's$ on U_1 and U_2 respectively.

Let $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ be an identity function, then h_P is $\mathcal{PF}\mathfrak{N}\delta$ $\mathcal{S}Cts$ but not $\mathcal{PF}\mathfrak{N}\delta\mathcal{S}Irr$, because the set $B_{\mathcal{PF}\mathfrak{N}}(A_2)^c$ is a $\mathcal{PF}\mathfrak{N}\delta\mathcal{S}os$ in U_2 but $h_P^{-1}(B_{\mathcal{PF}\mathfrak{N}}(A_2)^c) = B_{\mathcal{PF}\mathfrak{N}}(A_2)^c$ is not $\mathcal{PF}\mathfrak{N}\delta\mathcal{S}os$ in U_1.

Example 4.3. Let $U_1 = \{s_1, s_2, s_3, s_4\}$, $U_2 = \{t_1, t_2, t_3, t_4\}$ are the universe sets and the equivalence relations are $U_1/R = \{\{s_1, s_4\}, \{s_2\}, \{s_3\}\}$ and $U_2/R = \{\{t_1, t_4\}, \{t_2\}, \{t_3\}\}$. Let $A_1 = \left\{ \left\langle \frac{s_1}{0.3,0.1} \right\rangle, \left\langle \frac{s_2}{0.1,0.5} \right\rangle, \left\langle \frac{s_3}{0.2,0.45} \right\rangle, \left\langle \frac{s_4}{0.4,0.25} \right\rangle \right\}$ and $A_2 = \left\{ \left\langle \frac{t_1}{0.9,0.3} \right\rangle, \left\langle \frac{t_2}{0.5,0.7} \right\rangle, \left\langle \frac{t_3}{0.3,0.5} \right\rangle, \left\langle \frac{t_4}{0.6,0.7} \right\rangle \right\}$ be a Pythagorean fuzzy subsets of U_1 and U_2 respectively.

$$\underline{\mathcal{PF}\mathfrak{N}}(A_1) = \left\{ \left\langle \frac{s_1, s_4}{0.3, 0.25} \right\rangle, \left\langle \frac{s_2}{0.1, 0.5} \right\rangle, \left\langle \frac{s_3}{0.2, 0.45} \right\rangle \right\},$$
$$\overline{\mathcal{PF}\mathfrak{N}}(A_1) = \left\{ \left\langle \frac{s_1, s_4}{0.4, 0.1} \right\rangle, \left\langle \frac{s_2}{0.1, 0.5} \right\rangle, \left\langle \frac{s_3}{0.2, 0.45} \right\rangle \right\},$$
$$B_{\mathcal{PF}\mathfrak{N}}(A_1) = \left\{ \left\langle \frac{s_1, s_4}{0.25, 0.3} \right\rangle, \left\langle \frac{s_2}{0.1, 0.5} \right\rangle, \left\langle \frac{s_3}{0.2, 0.45} \right\rangle \right\},$$
$$\underline{\mathcal{PF}\mathfrak{N}}(A_2) = \left\{ \left\langle \frac{t_1, t_4}{0.6, 0.7} \right\rangle, \left\langle \frac{t_2}{0.5, 0.7} \right\rangle, \left\langle \frac{t_3}{0.3, 0.5} \right\rangle \right\},$$
$$\overline{\mathcal{PF}\mathfrak{N}}(A_2) = \left\{ \left\langle \frac{t_1, t_4}{0.9, 0.3} \right\rangle, \left\langle \frac{t_2}{0.5, 0.7} \right\rangle, \left\langle \frac{t_3}{0.3, 0.5} \right\rangle \right\},$$

$$B_{\mathcal{PFN}}(A_2) = \left\{ \left\langle \frac{t_1, t_4}{0.7, 0.6} \right\rangle, \left\langle \frac{t_2}{0.5, 0.7} \right\rangle, \left\langle \frac{t_3}{0.3, 0.5} \right\rangle \right\}.$$

Here $\tau_p(A_1) = \{0_P, 1_P, \underline{\mathcal{PFN}}(A_1), \overline{\mathcal{PFN}}(A_1), B_{\mathcal{PFN}}(A_1)\}$ and $\tau_p(A_2) = \{0_P, 1_P, \underline{\mathcal{PFN}}(A_2), \overline{\mathcal{PFN}}(A_2), B_{\mathcal{PFN}}(A_2)\}$ are the $\mathcal{PFN}ts's$ on U_1 and U_2 respectively. Let $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ be an identity function, then h_P is $\mathcal{PFN}\delta\mathcal{P}Cts$ but not $\mathcal{PFN}\delta\mathcal{P}Irr$, because the set

$$B_1 = \left\{ \left\langle \frac{s_1, s_4}{0.25, 0.35} \right\rangle, \left\langle \frac{s_2}{0.25, 0.45} \right\rangle, \left\langle \frac{s_3}{0.3, 0.2} \right\rangle \right\}$$

is a $\mathcal{PFN}\delta\mathcal{P}os$ in U_2 but $h_P^{-1}(B_1) = B_1$ is not $\mathcal{PFN}\delta\mathcal{P}os$ in U_1.

Example 4.4. Let $U_1 = \{s_1, s_2, s_3, s_4\}$, $U_2 = \{t_1, t_2, t_3, t_4\}$ are the universe sets and the equivalence relations are $U_1/R = \{\{s_1, s_4\}, \{s_2\}, \{s_3\}\}$ and $U_2/R = \{\{t_1, t_4\}, \{t_2\}, \{t_3\}\}$. Let $A_1 = \left\{ \left\langle \frac{s_1}{0.3,0.1} \right\rangle, \left\langle \frac{s_2}{0.1,0.5} \right\rangle, \left\langle \frac{s_3}{0.2,0.45} \right\rangle, \left\langle \frac{s_4}{0.4,0.25} \right\rangle \right\}$ and $A_2 = \left\{ \left\langle \frac{t_1}{0.9,0.3} \right\rangle, \left\langle \frac{t_2}{0.5,0.7} \right\rangle, \left\langle \frac{t_3}{0.3,0.5} \right\rangle, \left\langle \frac{t_4}{0.6,0.7} \right\rangle \right\}$ be a Pythagorean fuzzy subsets of U_1 and U_2 respectively.

$$\underline{\mathcal{PFN}}(A_1) = \left\{ \left\langle \frac{s_1, s_4}{0.3, 0.25} \right\rangle, \left\langle \frac{s_2}{0.1, 0.5} \right\rangle, \left\langle \frac{s_3}{0.2, 0.45} \right\rangle \right\},$$

$$\overline{\mathcal{PFN}}(A) = \left\{ \left\langle \frac{s_1, s_4}{0.4, 0.1} \right\rangle, \left\langle \frac{s_2}{0.1, 0.5} \right\rangle, \left\langle \frac{s_3}{0.2, 0.45} \right\rangle \right\},$$

$$B_{\mathcal{PFN}}(A) = \left\{ \left\langle \frac{s_1, s_4}{0.25, 0.3} \right\rangle, \left\langle \frac{s_2}{0.1, 0.5} \right\rangle, \left\langle \frac{s_3}{0.2, 0.45} \right\rangle \right\}$$

$$\underline{\mathcal{PFN}}(A_2) = \left\{ \left\langle \frac{t_1, t_4}{0.6, 0.7} \right\rangle, \left\langle \frac{t_2}{0.5, 0.7} \right\rangle, \left\langle \frac{t_3}{0.3, 0.5} \right\rangle \right\},$$

$$\overline{\mathcal{PFN}}(A_2) = \left\{ \left\langle \frac{t_1, t_4}{0.9, 0.3} \right\rangle, \left\langle \frac{t_2}{0.5, 0.7} \right\rangle, \left\langle \frac{t_3}{0.3, 0.5} \right\rangle \right\},$$

$$B_{\mathcal{PFN}}(A_2) = \left\{ \left\langle \frac{t_1, t_4}{0.7, 0.6} \right\rangle, \left\langle \frac{t_2}{0.5, 0.7} \right\rangle, \left\langle \frac{t_3}{0.3, 0.5} \right\rangle \right\}.$$

Here $\tau_p(A_1) = \{0_P, 1_P, \underline{\mathcal{PFN}}(A_1), \overline{\mathcal{PFN}}(A_1), B_{\mathcal{PFN}}(A_1)\}$ and $\tau_p(A_2) = \{0_P, 1_P, \underline{\mathcal{PFN}}(A_2), \overline{\mathcal{PFN}}(A_2), B_{\mathcal{PFN}}(A_2)\}$ are the $\mathcal{PFN}ts's$ on U_1 and U_2 respectively. Let $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ be an identity function, then h_P is $\mathcal{PFN}\delta\beta Cts$ but not $\mathcal{PFN}\delta\beta Irr$, because the set

$$C_1 = \left\{ \left\langle \frac{s_1, s_4}{0.1, 0.4} \right\rangle, \left\langle \frac{s_2}{0.2, 0.4} \right\rangle, \left\langle \frac{s_3}{0.3, 0.2} \right\rangle \right\}$$

is a $\mathcal{PFN}\delta\beta os$ in U_2 but $h_P^{-1}(C_1) = C_1$ is not $\mathcal{PFN}\delta\beta os$ in U_1.

Definition 4.2. *A $\mathcal{PFN}ts$ $(U_1, \tau_P(A_1))$ is known as a Pythagorean fuzzy nano $\delta\mathcal{S}U_{\frac{1}{2}}$ (resp. $\delta\mathcal{P}U_{\frac{1}{2}}$, $\delta\alpha U_{\frac{1}{2}}$ and $\delta\beta U_{\frac{1}{2}}$) (in short, $\mathcal{PFN}\delta\mathcal{S}U_{\frac{1}{2}}$ (resp. $\mathcal{PFN}\delta\mathcal{P}U_{\frac{1}{2}}$, $\mathcal{PFN}\delta\alpha U_{\frac{1}{2}}$ and $\mathcal{PFN}\delta\beta U_{\frac{1}{2}}$))-space, if each $\mathcal{PFN}\delta\mathcal{S}os$ (resp. $\mathcal{PFN}\delta\mathcal{P}os$, $\mathcal{PFN}\delta\alpha os$ and $\mathcal{PFN}\delta\beta os$) in X is $\mathcal{PFN}os$ in U_1.*

Theorem 4.2. *Let $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ be a $\mathcal{PFN}\delta Irr$ (resp. $\mathcal{PFN}\delta\mathcal{S}\ Irr$, $\mathcal{PFN}\delta\mathcal{P}Irr$, $\mathcal{PFN}\delta\alpha Irr$ and $\mathcal{PFN}\delta\beta Irr$) map. Then h_P is a $\mathcal{PFN}Cts$ map if X is a $\mathcal{PFN}\delta U_{\frac{1}{2}}$ (resp. $\mathcal{PFN}\delta\mathcal{S}U_{\frac{1}{2}}$, $\mathcal{PFN}\delta\mathcal{P}U_{\frac{1}{2}}$, $\mathcal{PFN}\delta\alpha U_{\frac{1}{2}}$ and $\mathcal{PFN}\delta\beta U_{\frac{1}{2}}$)-space.*

Proof. (i) Consider a $\mathcal{PFN}\delta os$ K in U_2. Therefore $h_P^{-1}(K)$ is a $\mathcal{PFN}\delta os$ in U_1. Since U_1 is a $\mathcal{PFN}\delta U_{\frac{1}{2}}$-space, $h_P^{-1}(K)$ is a $\mathcal{PFN}os$ in U_1. Hence h_P is a $\mathcal{PFN}Cts$ map.

(ii) Consider a $\mathcal{PFN}os$ K in U_2. Then K is a $\mathcal{PFN}\delta\mathcal{S}os$ in U_2. Therefore $h_P^{-1}(K)$ is a $\mathcal{PFN}\delta\mathcal{S}os$ in U_1. Since U_1 is a $\mathcal{PFN}\delta\mathcal{S}U_{\frac{1}{2}}$-space, $h_P^{-1}(K)$ is a $\mathcal{PFN}os$ in U_1. Hence h_P is a $\mathcal{PFN}Cts$ map.

(iii) Consider a $\mathcal{PFN}os$ K in U_2. Then K is a $\mathcal{PFN}\delta\mathcal{P}os$ in U_2. Therefore $h_P^{-1}(K)$ is a $\mathcal{PFN}\delta\mathcal{P}os$ in U_1. Since X is a $\mathcal{PFN}\delta\mathcal{P}U_{\frac{1}{2}}$-space, $h_P^{-1}(K)$ is a $\mathcal{PFN}os$ in U_1. Hence h_P is a $\mathcal{PFN}Cts$ map.

(iv) Consider a $\mathcal{PFN}os$ K in U_2. Then K is a $\mathcal{PFN}\delta\alpha os$ in U_2. Therefore $h_P^{-1}(K)$ is a $\mathcal{PFN}\delta\alpha os$ in U_1. Since U_1 is a $\mathcal{PFN}\delta\alpha U_{\frac{1}{2}}$-space, $h_P^{-1}(K)$ is a $\mathcal{PFN}os$ in U_1. Hence h_P is a $\mathcal{PFN}Cts$ map.

(v) Consider a $\mathcal{PFN}os$ K in U_2. Then K is a $\mathcal{PFN}\delta\beta os$ in U_2. Therefore $h_P^{-1}(K)$ is a $\mathcal{PFN}\delta\beta os$ in U_1. Since U_1 is a $\mathcal{PFN}\delta\beta U_{\frac{1}{2}}$-space, $h_P^{-1}(K)$ is a $\mathcal{PFN}os$ in U_1. Hence h_P is a $\mathcal{PFN}Cts$ map.

Theorem 4.3. *Let $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ and $g_P : (U_2, \tau_P(A_2)) \to (U_3, \tau_P(A_3))$ be $\mathcal{PFN}\delta Irr$ (resp. $\mathcal{PFN}\delta\mathcal{S}Irr$, $\mathcal{PFN}\delta\mathcal{P}Irr$, $\mathcal{PFN}\delta\alpha Irr$ and $\mathcal{PFN}$ $\delta\beta Irr$) maps, then $g_P \circ h_P : (U_1, \tau_P(A_1)) \to (U_3, \tau_P(A_3))$ is a $\mathcal{PFN}\delta Irr$ (resp. $\mathcal{PFN}\delta\mathcal{S}Irr$, $\mathcal{PFN}\delta\mathcal{P}Irr$, $\mathcal{PFN}\delta\alpha Irr$ and $\mathcal{PFN}\delta\beta Irr$) map.*

Proof. Consider a $\mathcal{PFN}\delta os$ K in U_3. So $g_P^{-1}(K)$ is a $\mathcal{PFN}\delta os$ in U_2. As h_P is a $\mathcal{PFN}\delta Irr$ map, $h_P^{-1}(g_P^{-1}(K))$ is a $\mathcal{PFN}\delta os$ in U_1. Thus $g_P \circ h_P$ is a $\mathcal{PFN}\delta Irr$ map. The other cases are similar.

Theorem 4.4. *Consider a $\mathcal{PFN}\delta Irr$ (resp. $\mathcal{PFN}\delta\mathcal{S}Irr$, $\mathcal{PFN}\delta\mathcal{P}Irr$, $\mathcal{PFN}\delta\alpha Irr$ and $\mathcal{PFN}\delta\beta Irr$) map $h_P : (U_1,\ \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ and a $\mathcal{PFN}\delta Cts$ (resp. $\mathcal{PFN}\delta\mathcal{S}Cts$, $\mathcal{PFN}\delta\mathcal{P}Cts$, $\mathcal{PFN}\delta\alpha Cts$ and $\mathcal{PFN}\delta\beta Cts$) map $g_P : (U_2, \tau_P(A_2)) \to (U_3, \tau_P(A_3))$. Then $g_P \circ h_P : (U_1, \tau_P(A_1)) \to (U_3, \tau_P(A_3))$ is a $\mathcal{PFN}\delta Cts$ (resp. $\mathcal{PFN}\delta\mathcal{S}Cts$, $\mathcal{PFN}\delta\mathcal{P}Cts$, $\mathcal{PFN}\delta\alpha Cts$ and $\mathcal{PFN}\delta\beta Cts$) map.*

Proof. Consider a $\mathcal{PF}\mathfrak{N}os$ K in U_3. So $g_P^{-1}(K)$ is a $\mathcal{PF}\mathfrak{N}\delta os$ in U_2. As h_P is a $\mathcal{PF}\mathfrak{N}\delta Irr$ map, $h_P^{-1}(g_P^{-1}(U))$ is a $\mathcal{PF}\mathfrak{N}\delta os$ in U_1. Thus $g_P \circ h_P$ is a $\mathcal{PF}\mathfrak{N}\delta Cts$ map. The other cases are similar.

Theorem 4.5. *Consider a map $h_P : (U_1, \tau_P(A_1)) \to (U_2, \tau_P(A_2))$ from a $\mathcal{PF}\mathfrak{N}ts$ U_1 into a $\mathcal{PF}\mathfrak{N}ts$ U_2. The following are equivalent if U_1 and U_2 are $\mathcal{PF}\mathfrak{N}\delta\beta U_{\frac{1}{2}}$-spaces.*

(i) h_P is a $\mathcal{PF}\mathfrak{N}\delta\beta Irr$ map.

(ii) $h_P^{-1}(K)$ is a $\mathcal{PF}\mathfrak{N}\delta\beta cs$ in U_1 for every $\mathcal{PF}\mathfrak{N}\delta\beta cs$ K in U_2.

(iii) $\mathcal{PF}\mathfrak{N}cl(h_P^{-1}(K)) \subseteq h_P^{-1}(\mathcal{PF}\mathfrak{N}cl(K))$ for every pfs K of U_2.

Proof. (i) $\to$ (ii): Consider a $\mathcal{PF}\mathfrak{N}\delta\beta cs$ K in U_2. It follows K^c is a $\mathcal{PF}\mathfrak{N}\delta\beta os$ in U_2. As h_P is $\mathcal{PF}\mathfrak{N}\delta\beta Irr$, $h_P^{-1}((K)^c)$ is a $\mathcal{PF}\mathfrak{N}\delta\beta os$ in U_1. We know that $h_P^{-1}((K)^c) = \left(h_P^{-1}(K)\right)^c$. Hence $h_P^{-1}(K)$ is a $\mathcal{PF}\mathfrak{N}\delta\beta cs$ in U_1.
(ii) $\to$ (iii): Consider a $\mathcal{PF}s$ K in U_2 and $K \subseteq \mathcal{PF}\mathfrak{N}\delta\beta cl(K)$. Then $h_P^{-1}(K) \subseteq h_P^{-1}(\mathcal{PF}\mathfrak{N}\delta\beta cl(K))$. Since $\mathcal{PF}\mathfrak{N}\delta\beta cl(K))$ is a $\mathcal{PF}\mathfrak{N}\delta\beta cs$ in U_2. By presumption, $h_P^{-1}((\mathcal{PF}\mathfrak{N}\delta\beta cl(K)))$ is a $\mathcal{PF}\mathfrak{N}\delta\beta cs$ in U_1. Also, as U_1 is $\mathcal{PF}\mathfrak{N}\delta\beta U_{\frac{1}{2}}$-space, $h_P^{-1}(\mathcal{PF}\mathfrak{N}\delta\beta cl(K))$ is a $\mathcal{PF}\mathfrak{N}\delta\beta cs$ in U_1.
(iii) $\to$ (i): Consider a $\mathcal{PF}\mathfrak{N}\delta\beta cs$ K in U_2. As U_2 is $\mathcal{PF}\mathfrak{N}\delta\beta U_{\frac{1}{2}}$-space, K is $\mathcal{PF}\mathfrak{N}cs$ in U_2 and $\mathcal{PF}\mathfrak{N}cl(K) = (K)$. Thus $h_P^{-1}(K) = h_P^{-1}(\mathcal{PF}\mathfrak{N}\delta\beta cl(K)) \supseteq \mathcal{PF}\mathfrak{N}\delta\beta cl(h_P^{-1}(K)) = \mathcal{PF}\mathfrak{N}cl(h_P^{-1}(K))$. But clearly $(h_P^{-1}(K)) \subseteq \mathcal{PF}\mathfrak{N}cl(h_P^{-1}(K))$. Therefore $\mathcal{PF}\mathfrak{N}cl((h_P^{-1}(K))) = h_P^{-1}(K)$. It follows $h_P^{-1}(K)$ is a $\mathcal{PF}\mathfrak{N}cs$ and so it is a $\mathcal{PF}\mathfrak{N}\delta\beta cs$ in U_1. Hence h_p is $\mathcal{PF}\mathfrak{N}\delta\beta irr$ map. The proof is similar for other cases of $\mathcal{PF}\mathfrak{N}\delta os$, $\mathcal{PF}\mathfrak{N}\delta\mathcal{P}os$, $\mathcal{PF}\mathfrak{N}\delta\mathcal{S}os$ and $\mathcal{PF}\mathfrak{N}\delta\alpha os$.

5. Application

Measures of similarity is a real-valued function which quantifies the similarity between two pythagoren fuzzy sets and its value is always expressed as a number between 0 and 1: 0, a low level of similarity, and 1, a high level of similarity.

Example 5.1. In tandem with economic development, it will unavoidably lead to water pollution, which will sooner or later endanger humans themselves. The treatment of water pollution has become a critical environmental and social issue.In particular, Cauvery river, a largest river in Tamil Nadu (India), is polluted by the dying factories and industrial wastewater, and the effect is getting severe year by year. On one hand, industrial wastewater discharge from Tirupur, Bhavani, Erode and Karur district gradually reached emission standards. Conversely, the infrastructure and capacity of wastewater treatment facilities are also being continually optimized. In order to maintain the stability of wastewater treatment capacity, the factories need to update the facilities periodically.

Recently, a dying factory need to renew batch of wastewater treatment facilities. Multi criteria decision making problem of an effective alternative treatment can be selected from five alternatives treatments A_1, A_2, A_3, A_4 and A_5 based on four expected criteria C_1, C_2, C_3 and C_4 (expense, Installation time, Renewal Period/durability and manpower respectively). Due to existence of complexity and uncertain in decision making Pythagorean fuzzy set is more convenient for decision makers expressing uncertain informations. The following algorithm will be self explanatory for the process of MCDM

Algorithm:

Step 1: Frame the pfs of each alternative treatments with four criteria as universe of discourse.

Step 2: An ideal set $I = \{< I, C_1; 1.0, 0.0 >, < I, C_2; 1.0, 0.0 >, < I, C_3; 1.0, 0.0 >, < I, C_4; 1.0, 0.0 >\}$ refers the pfs which completely fulfils the expectation of each criteria.

Step 3: Identify the $Zhang$ similarity measure between the pfs I and each alternative treatments.

Step 4: Choose the alternative treatment which has the $Zhang$ similarity measure very close to 1.

Table 1. Pythagorean fuzzy sets of each alternates

	Criteria 1 (C_1)	Criteria 2 (C_2)	Criteria 3 (C_3)	Criteria 4 (C_4)
A_1	$< A_1, C_1; 0.8, 0.4 >$	$< A_1, C_2; 0.3, 0.8 >$	$< A_1, C_3; 0.6, 0.3 >$	$< A_1, C_4; 0.6, 0.4 >$
A_2	$< A_2, C_1; 0.4, 0.7 >$	$< A_2, C_2; 0.3, 0.8 >$	$< A_2, C_3; 0.5, 0.6 >$	$< A_2, C_4; 0.6, 0.5 >$
A_3	$< A_3, C_1; 0.6, 0.5 >$	$< A_3, C_2; 0.7, 0.4 >$	$< A_3, C_3; 0.3, 0.8 >$	$< A_3, C_4; 0.3, 0.5 >$
A_4	$< A_4, C_1; 0.7, 0.4 >$	$< A_4, C_2; 0.4, 0.7 >$	$< A_4, C_3; 0.5, 0.7 >$	$< A_4, C_4; 0.6, 0.4 >$
A_5	$< A_5, C_1; 0.9, 0.2 >$	$< A_5, C_2; 0.4, 0.8 >$	$< A_5, C_3; 0.6, 0.3 >$	$< A_5, C_4; 0.7, 0.2 >$

Clearly, all values in the Table 1 are pfs's. Now we calculate the Zhang similarity measure $S_Z(I, A_i)$ of each alternative treatments.

Table 2. Zhang similarity measure of each item through criteria.

	A_1	A_2	A_3	A_4	A_5
$\mathcal{S}_Z(I, A_i)$	0.535	0.415	0.474	0.493	**0.594**

From Table 2, it is clear that $S_Z(I, A_2) < S_Z(I, A_3) < S_Z(I, A_4) < S_Z(I, A_1) < S_Z(I, A_5)$.

Finally it is concluded that the alternative treatment A_5 is more efficient.

6. Conclusion

In this paper, $\mathcal{PFN}\delta Cts$, $\mathcal{PFN}Cts$, $\mathcal{PFN}\delta\mathcal{S}Cts$, $\mathcal{PFN}\delta\mathcal{P}Cts$, $\mathcal{PFN}\delta\alpha Cts$, and $\mathcal{PFN}\delta\beta Cts$ respective irresolute map is defined using $\mathcal{PFN}\delta o$, $\mathcal{PFN}\delta\mathcal{S}o$, $\mathcal{PFN}\delta\mathcal{P}o$, $\mathcal{PFN}\delta\alpha o$ and $\mathcal{PFN}\delta\beta o$ set and its properties are analyzed with the examples. Then Pythagorean fuzzy continuous maps are compared with other generalized Pythagorean fuzzy continuous maps. Also we extended the concept of Pythagorean fuzzy irresolute maps in Pythagorean fuzzy topological spaces using $\mathcal{PFN}o$ sets. Some examples and basic relationships between the mappings were also discussed. In future, these can be extended to Pythagorean fuzzy open, closed, homeomorphism and contra maps. Application for MCDM to the real world problem was solved with the proposed similarity measure. In future, MCDM to the real world problem can be developed to the $\mathcal{PFN}ts$.

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